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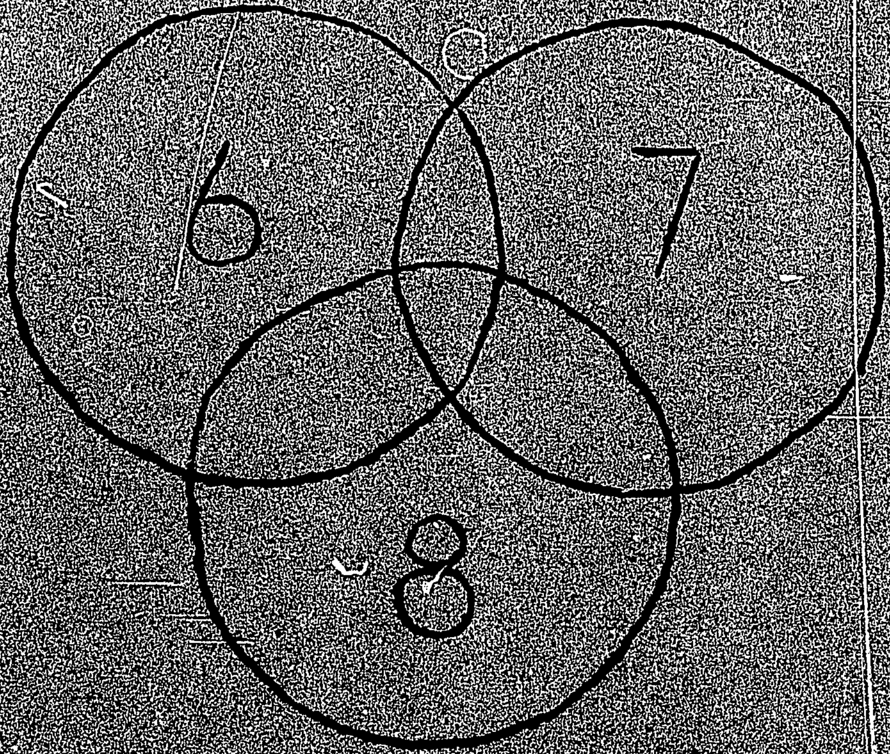
ABSTRACT

This curriculum guide outlines the mathematical content which whould be presented in grades 6, 7, and 8 - the new middle school organization. It is the result of a study made by a committee of principals and teachers of the needs and characteristics of adolescents. The following are included for each of the content areas: (1) topics which may be studied, (2) skills, (3) concepts, (4) application, (5) suggestions for implementation, and (6) helpful material and references. The mathematical content is contained within the conventional junior high school curriculum: number concepts, sets, algebra, geometry, statistics and probability, and numerical trigonometry. Also included are the objectives and philosophy of the school system. (RS)

Transfer: ERIC/...

MATHEMATICS

CURRICULUM GUIDE FOR GRADES



Joliet Public Schools
School District No. 86, Will County
Joliet, Illinois

CURRICULUM GUIDE FOR THE

MATHEMATICS

FOR JUNIOR HIGH, GRADES 6-7-8

DR. DONALD J. D'AMICO

Superintendent of Schools

District 86

BOARD OF SCHOOL INSPECTORS

City of Joliet

F O R E W O R D

The Joliet Public Schools have been committed to the concept of the Junior High school since the early part of the century. This school organization makes possible varied program and instructional approaches, which can best serve the educational needs of emerging adolescents. To the present time the School District has been structured according to a K-6-2 plan with only 7th and 8th grades in the Junior High. With the advent of the 1970-71 school year, the District will begin implementation of a new K-5-3 plan, which incorporates 6th grade into the Junior High school program.

This reorganization was undertaken by the Board of School Inspectors after considering evolving psychological and social characteristics of contemporary society and recent research findings concerning adolescent youth. While alteration of the Junior High school grade structure to include 6th grade is not uncommon, the District has not adopted all the changes associated with such reorganization. For instance, the District chose to call this pattern "Junior High" not "Middle School". The programs of the Joliet Junior High schools will remain rooted in the sound philosophical bases which have been developed for early adolescent education over the past half-century. Consequently, to change the school name to Middle School would be to make a distinction without a difference.

In preparation for these changes it was deemed necessary that there be a thorough study of curriculums and programs. It was essential that curriculum programs be oriented to the needs, characteristics, and idiosyncrasies of pubescent youth. Further, that program objectives and functions be evaluated in terms of their implications for effecting sound instructional learning.

Such a study was begun in the Fall of 1968 with the appointment of a representative committee, consisting of the Junior High and Elementary principals of the District and outstanding teachers from the various subject areas and sixth grade.

The District gratefully acknowledges the contributions made to the work of this committee by:

Mary Louise Condon, Principal
Hufford I, Junior High

John Godde, Principal
Hufford II, Junior High

Nadine Ewing, Principal
Taft School

Jack Moss, Principal
Washington Junior High

John O'Hara, Principal
Forest Park School

Dorothy Ryan, Principal
McKinley Park School

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Gompers, Language Arts

Jacob Broncato, Director
Learning Resource Center

Frank Fagen
Thompson School

Margaret Hills
Raynor Park School

Florence Loeffler
Hufford II, Speech

Juanita Meunier
Hufford, Physical Education

William Reed
Washington, Social Studies

Margarette Shields
Pershing School

Special acknowledgement is made of the contribution which Dr. R. Jerry Cantlon, Department of Education, Illinois State University has made to this Project. His objective assistance and encouragement was invaluable as a resource consultant.

Dale L. Lang
Assistant Superintendent
For Instruction

ACKNOWLEDGEMENT

This curriculum guide is dedicated to the students of District #86.

Throughout the discussion and writing of this program the students' intellectual welfare was uppermost in the minds of the committee.

A thank you is in order to:

Clariann Woolard - Gompers
Arlene Hambaugh - Hufford
James Lauretig - Washington
Florence Nadelhoffer - Pershing
Warren Upchurch - Eliza Kelly
John O'Hara - Forest Park

who put into this guide countless hours of thought, discussion and writing.

A special thank you to Mr. W. Banks, our consultant, who guided us through the mass of mathematics curriculum material enabling us to identify the needs of our children and society, and blending these facets into a working, practical curriculum guide.

TABLE of CONTENTS

JUNIOR HIGH PHILOSOPHY OF THE JOLIET PUBLIC SCHOOLS

The Joliet Junior High Schools are institutions whose character and purpose are dictated by the unique nature of the pupils it serves-the emerging adolescent. In order to most effectively fulfill their purposes as found in the objectives of education and the functions of the junior high schools, the Joliet Junior High Schools:

1. are oriented to the concept that the pupil's total school experience comprehends more than the organized educational activities of the school
2. are established as 3-year schools incorporating grades 6, 7, & 8
3. are organized for Cooperative Team Teaching within a structure of flexible scheduling
4. utilize the Broad Fields Curriculum structure
5. are committed to Teaching-Learning strategies which are based fundamentally upon the individual learner, the nature of learning, and the present and future relation of the pupil to his society
6. will include a regularly scheduled period of time to areas of student interest and needs which are supportive of the broad field curriculum

The Joliet Junior High Schools, assuming a measure of responsibility which is consistent with the school's most effective role in society, are committed to the achievement of the following broad objectives:

to expand and enrich the progression of learning begun within the general educational framework of the elementary school to develop within students their abilities to observe, listen, read, calculate, think, speak, and write with purpose and comprehension

to create opportunities for learning consistent with the student's intellectual capacity through the establishment of the most desirable instructual, programmatic, and environmental features

to provide those experiences which are necessary for the appropriate growth of the individual student relative to his mental, emotional, physical, and social development

to organize experience opportunities assuring a smoother transition from one educational level to another

to develop respect for the cultural contributions of all men

to make available sufficient personal interest explorations

to initiate a clear understanding of the industry and culture of the adult world

to assist students in meeting the challenges of adolescence through extensive guidance

to provide opportunities for enriched living which come from appreciation of and expression in the arts

to provide experiences designed to develop appropriate attitudes and values necessary for living in a democracy

The Joliet Junior High Schools are oriented such as to pursue the achievement of these objectives through what is known of the "Functions" of this level of education, through implementation of their Curriculum, and by what is believed to be the most effective and efficient application of human and material resources available for an educational program in this community.

THE JUNIOR HIGH SCHOOL FUNCTIONS: AN INTRODUCTION

In the foregoing portion of this document there is presented a statement of philosophy which by its nature describes in general language the character and ultimate objectives of the Joliet Junior High Schools. This statement is intended to provide a valid basis for the development of the most desirable educational program possible for these schools.

Concepts which are more specific are now required for the development and implementation of such a program. These are commonly referred to as functions, which themselves may be both proximate aims of education at this level and responsibilities of the junior high school for providing those conditions or elements which promise to lead most directly to the achievement of the ultimate objectives of education.

The following comprehensive statement of functions has been adopted by the Joliet Junior High Schools to indicate the responsibilities which have been accepted as being most important for the fulfillment of the objectives of junior high school education.

STATEMENT OF FUNCTIONS OF THE JOLIET JUNIOR HIGH SCHOOLS

Articulation

Is the provision which is made to help students achieve a satisfying transition from programs and experiences encountered during their childhood years to a program of educative experience consistent with their needs during early, middle, and late adolescence.

Must be a concern of the curricular, instructional, guidance, administrative, extracurricular, and school-community relations programs of the school.

Must be concerned for the continuity of student devolopment between consecutive learning experiences and from one school level to another---from elementary to junior high, within the junior high, and from junior high to senior high school.

Requires concern for the maximum growth of students in the full range of their characteristics, e.g. mental, emotional, cultural, social, and physical.

Is the product of the continuous individual and collective efforts of all professionals concerned---teachers, supervisors, counselors, administrators, and other specialists.

Exploration

Is that provision which is made to stimulate and enhance the puils' regard for learning through that questing which is satisfying to their developmental needs in the educational, vocational, avocational, social, civic, recreational, cultural, and ethical aspects of individual growth and maturation.

Must be a common element of the total school program, constituting one of its central purposes, and should be an outcome specifically of the curriculum, instruction, and the unstructured educational endeavor of the total school environment.

To serve students' needs, exploratory activity should be motivational, purposeful, and enlightening.

Should contribute toward the identification and investigation of student aptitudes, interests, and capacities, thereby serving as a basis for effective instructional and curricular leadership, educational and prevocational guidance and goal-setting, and the individual student's self-identification, self-evaluation, and self-realization.

Should provide for the extension of previous learnings as well as the acquisition of new learnings consistent with worthwile educational goals and the likely needs of individual pupils.

Should provide educative experiences which will stimulate the growth of students in social, cultural, civic, ethical, and avocational appreciations and ambitions.

Should enable students to pursue answers to those fundamental questions of life which characteristically arise at this age having to do with the nature of the world and universe, the self, the individual's place in a dynamic society, his personal values, and his broad goals in life.

Guidance

Is an educational process which emanates from individual and group interaction between students and their teachers, the guidance staff, other adult leaders, and their peers.

Is fundamentally for the purpose of assisting the individual to know himself and to help him interact with his environment at the highest possible level of effectiveness, both presently and in the future.

Should be personal, social, emotional, vocational, educational, and intellectual in its character, with each of these being mutually reinforcing as they relate to the individual and the group needs of early adolescents.

Should serve the individual and group needs of students through three major roles--the developmental, the preventive, and the remedial.

Should be centered in the program of general education, with all elements of the school's educational and student services programs contributing to the achievement of guidance goals.

Is inherent in good teaching, which assumes a commitment to the individualization of instruction.

Is contingent upon having individual students well known by at least one of their teachers, having a continuing guidance relationship between every pupil and a guidance specialist, and having the presence of an adequate number of suitable adult role models in the school.

Should be the object of continuous process in behalf of its major objectives.

Should be related to the previous, present, and future educational and personal experiences of individual pupils and should provide for the continuous evaluation of those experiences by teachers, counselors, parents, and the students themselves.

Should be contributory to the student's learning and maturation with respect to:

--the acquisition and enhancement of basic skills, the development of understandings, and the stimulation of intellectual curiosity, consonant with his aptitudes and abilities

---his develelopment in and understanding of the importance of mental and physical health, especially as these relate to his rapidly changing physical self and his acquisition of appropriate sex roles

--his socialization, including the achievement of appropriate levels of individual self-discipline and group behavior

--his growth in esthetic appreciation and in moral, ethical and material values

--the identification of educational and vocational interests, and development of desirable attitudes and practices in the utilization of leisure time

--being accepting of the fact that they are different, are expected to be different, should be proud to be different, and have responsibilities in the light of these differences.

Integration

Is that academic endeavor which makes possible the student's understanding of the ways that elements of his environment and his being are or may be interrelated.

Develops competence in the utilization of processes required for:

--the explanation of phenomena perceived

--the interpretation of one's own behavior and values

--appreciation for the plurality of intellectual approaches which may be utilized in the pursuit of problem-solving

Should help the student to perceive that each learning experience contributes toward a new and staisfying unity of knowledge--such unity emanating from and contributing to the dynamism of his personal growth and to his interaction with his environment.

Should be inherent within the organization of the curriculum, planning for instruction, and teacher-student interaction.

May be inferred from the student's striving for a more meaningful organization of experience, his attainment of individual objectives, and the satisfaction which he derives from the attainment of such goals.

Differentiation

Each student is a unique being for whom there exists special present and ultimate goals of education.

The school is responsible for utilizing individualized means to facilitate the student's maximal development toward these goals, being consistent with the nature of his capacities and of our objectives of education for our junior high schools.

Teachers, counselors, administrators, and other specialists recognize and work with individual differences in the area of:

intellectual, physical, social, and emotional maturity
innate intellectual characteristics
physical and mental health
attitudes
achievements
work habits
values
cultural and family orientation

The school provides a blend of instructional, curricular, programmatic, and physical provisions and means, as well as those situations arising from the school environment which provides additonal opportunities for the achievement of this function.

Socialization

Is that provision which is made to assist the early adolescent in becoming an effective member of contemporary society and to help him establish the basis for desirable social development in both present and later stages of maturation.

Should emphasize the student's growth in:

--democratic philosophy and citizenship responsibility

--his ability to understand the complexities and the challenges of our pluralistic society

--his ability to identify and to apply critical thinking to societal problems

--his ability to give meaning and purpose to the idealism which is unique to his age

Should incorporate the following principles:

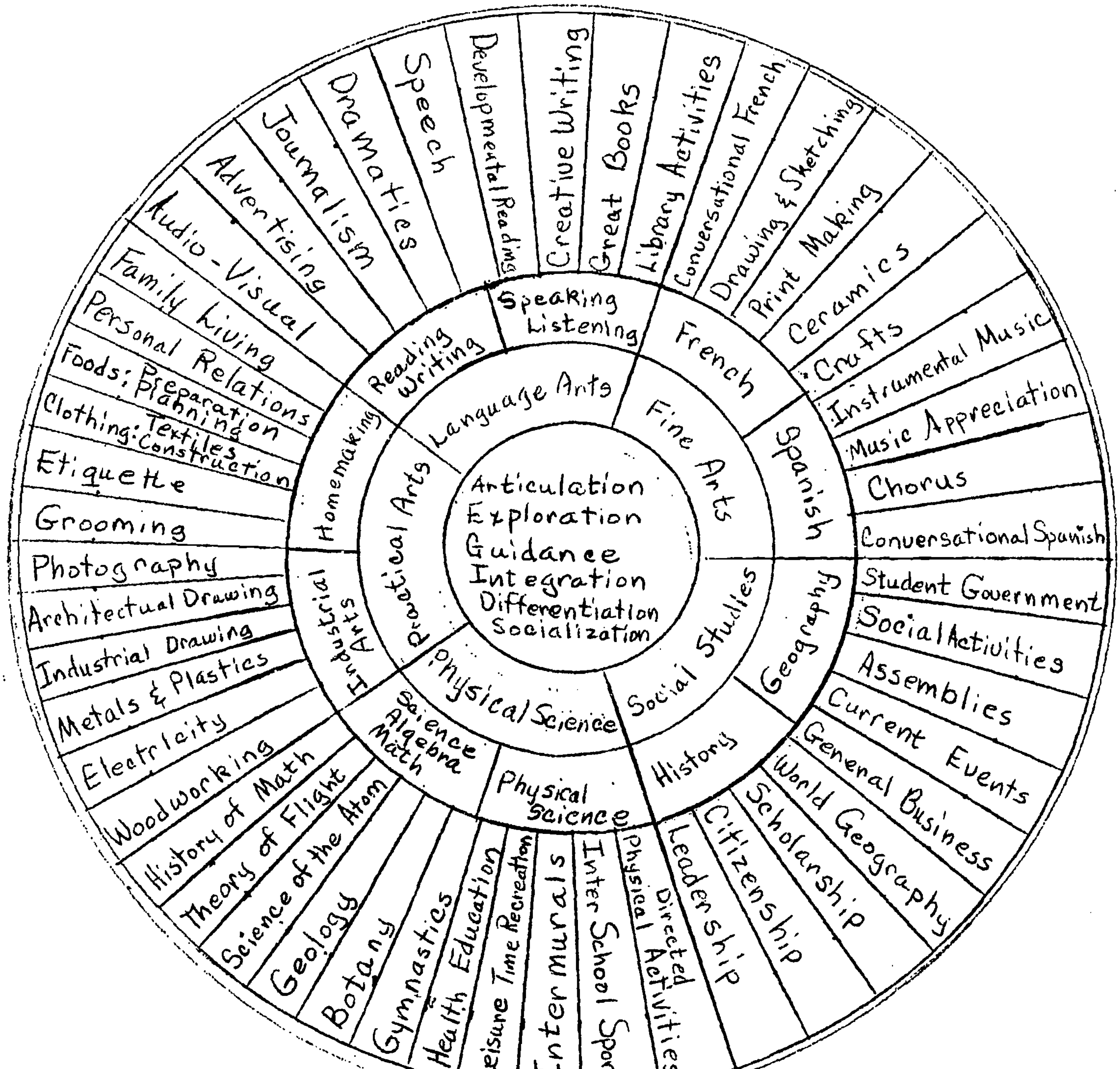
Fundamental to the achievement of this function, the objectives of which should be pursued in all areas of school life--the curriculum, classroom instruction, and the unstructured activity of the school-is the dynamic interrelationship of socialization factors operating within an environment conducive to the achievement of socialization goals.

Socialization should be the responsibility of concern of all members of the school staff--teachers, counselors, administrators, other specialists, and nonprofessionals--who, through a unity of effort, work in behalf of the socialization goals of the individual student and of all students in their group relationships.

Each student's progress toward socialization goals should be the subject of a continuing evaluation, involving all personnel concerned.

JOLIET JUNIOR HIGH SCHOOLS

A CURRICULUM MODEL



STATEMENT OF PRINCIPALS

The Joliet Junior High Schools are designated as team teaching schools. Team teaching in educational vocabulary has as many different interpretations as there are schools using this method. It is an arrangement wherby two or more teachers cooperatively plan, instruct, and evaluate class groups so as to take advantage of the special competencies of the team members.

A broad definition of team teaching must be used in order to include the programs now functioning in our district. Teachers should have the opportunity to intellectually explore team teaching and to develop their own programs. Human variability is very real and the range of variability is extremely great at the junior high level. Acceptance of these facts and recognition of their implications will produce a variety of programs to meet the needs of the junior high student.

The sucessful impact of team teaching upon the students is best realized when the abilities of each student, as well as the instructors, are considered. The staff acting as a team hold the responsibility, in this respect, of deciding what is to be taught to each child, by whom, when, and how, all in keeping with principles of mathematics and the philosophy of the school system. Although this places a huge responsibility and task upon the team, the development of each child's potential is only possible when such responsibilities are undertaken.

The committee does not intend this document to be restrictive but rather serve as a guide to what should be taught to students in sixth, seventh and eight grade. The sections which follow are not intended to be teaching units.

The order in which they appear and the order in which the topics and concepts are listed is not meant to suggest the order in which they are taught. It is the responsibility of the team to develope from this guide teaching units which are appropriate to the abilities of the students. Hence, as a guide, this document can be edited, revised, added to, subtracted from, and contracted at the judgement of the team.

Developing problem solving ability is a most important function of the school. In fact, in the large sense, this is the sole purpose of education - to enable students to solve their problems and society's problems. To give problem solving the attention it should have was impossible within the format of these guidelines. The usual written problems found in any math text are important. But other problems are equally important. These are the larger, longer range problems which preferably evolue from pupil needs and interests. Properly used, problems can motivate the learning of new concepts: they are also necessary to the complete understanding of concepts and skills.

No committee in a few weeks time is capable of creating curriculum guidelines which are satisfactory to everyone (including themselves). It is hoped, expected and necessary to improvement of instruction in the Joliet schools, that the mathematics staff criticize this document freely, offer suggestions for revision and that the administration provide for an early and periodic revision.

ALGEBRA

This curriculum is to provide an introduction to this subject and is not to be interpreted as an expanded program in algebra. We do not expect formal equation solving techniques to be developed; Understanding basic algebraic concepts is primary along with increased ability to write open sentences for word problems.

Topics Which May Be Studied.

Open sentences
Equations
Equivalent sentences
Inequalities
Variables and replacement set
Solution set
Solving equations
Positive and Negative numbers

Absolute value
Compound Open Sentences
Number line
Graphing
Exponents
Scientific notation
Functions and relations

Skills

Translating word sentences into number sentences.
Replacing the variable with a number of the replacement set to find the solution set.
Solving equations by inspection.
Simplyfying open sentences.
Using addition, subtraction, multiplication, and division.
Using properties of number systems to help solve number sentences.
Associating graphing and algebraic equations
Graphing on a number line or set of axes
Working with exponets

Concepts

An open sentence may be true or false.
An equation is a sentence in which two expressions represent the same number.
Equivalent equations
Inequalities
Equivalent sentences have the same solution set.
A variable is the symbol used to represent the unknown in an open sentence.
A variable is replaced by numbers of the replacement set.
A member of the solution set makes the statement true and must be in the replacement set.
Solution sets may contain positive and negative numbers.

Concepts (con't.)

Solution sets of compound open sentences involve intersection and union of sets.
A number line graph can be used to show the solution of one variable.
Axes are used to show solution sets of ordered pairs.
Exponents can be used to represent large numbers.
Negative exponents can be used to represent small numbers.

Application

Primarily, the student may be referred to previously learned principles and properties of numbers. The employment of algebra rests in a variety of subject areas. To point to the value of this employment, algebra may be applied to solving word problems involving one or more unknowns. Suggestively the thermometer and credit and debt may be utilized for positive and negative numbers; while graphing two or more variables is similar to finding a point on the globe.

Suggestions for Implementation

Games such as guess the function rule game can be used to help in understanding functions.
Students should not be expected to solve equations using formal techniques though some of them may recognize these techniques
Students should be encouraged to apply what they already know about numbers to solve equations.

Helpful Material and References

Introduction to Mathematics, Addison Wesley, pp. 175-232

Modern Mathematics Through Discovery II, Silver Burdett, pp. 292-336.

Modern School Mathematics, Houghton Mifflin, pp. 153-170, 257-330.

Growth in Arithmetic, Harcourt Brace, pp. 11-21, 27-58.

School Mathematics II, Addison Wesley, pp. 397-419.

GEOMETRY

The study of geometry at this level should be mostly intuitive and informal. Activities which develop spatial perception are important. Informal or oral proof should be encouraged and developed. At the very lease, concepts should be the main emphasis and not just the acquisition of vocabulary.

Topics Which May Be Studied

- Points
- Lines
- Planes
- Line segments
- Rays
- Angles
- Perpendiculars to lines and planes
- Parallel and skew lines and planes
- Open curves
- Simple closed curves
- Polygons (types)
- Triangles (types)
- Spatial figures
- Polyhedrons
- Congruence
- Similarity
- Symmetry
- Constructions
- Regions in a plane (triangular, rectangular, etc.)
- Drawings representing three dimensional objects

Skills

Recognizing symbols such as
Constructions with an unmarked straightedge and compass
Drawingswith protractor, ruler, compass and other precision instruments

Concepts

Point
Line
Plane
Through two points there is exactly one line.
Through three points not on a line there is exactly one plane.
Line segment
Ray
An angle is the union of two rays with a common end point
Parallel lines and planes
Lines intersect in a point, while planes intersect in a line.
Parallel Lines and planes
Perpendicular lines intersect at right angles.
Open and closed curves and the difference between them.
A circle consists of all the points in the plane at a given distance from the fixed point
A polyhedron is a closed surface formed by polygonal regions.
Congruency of geometric figures, angles and segments.
Similarity of geometric figures
Symmetry

Application

May work with constructions in conjunction with art, sewing, shop, and decorating.
Some practice in making scale drawings can come in connection with indirect measurement, trigonometry and proportion.
Show the use of symmetry in arranging furniture to balance a room, the symmetry in a leaf, flower, etc.
Make use of similar triangles in solving proportions.
Refer to sets for intersection and union and their application to geometry, statistical graphs and charts.

Suggestions for Implementation

Much of this geometry can and should be developed in connection with work in other areas such as statistics, measurement and trigonometry. Care should be given to keep clear that angles and triangles are unions of rays and segments and not regions in the plane.
Many visual aids should be used especially to help in space geometry.
In making formal constructions students should use only compass and unmarked straightedge. However, drawings of these demensions, objects bar graphs, circle graphs, etc. using other instruments should be given attention.

Helpful Material and References

School Mathematics I, Addison Wesley, pp. 73-83, 161-176, 261-270, 367-372.
School Mathematics II, Addison Wesley, pp. 71-84, 153-171, 263-272, 377-390.
Modern Mathematics Through Discovery I, Silver Burdett, pp. 211-300
Modern Mathematics Through Discovery II, Silver Purdett, pp. 135-207
Extending Mathematics, American Book Co., pp. 129-172, 321-365.
Introduction to Mathematics, Addison Wesley, pp. 243-282.
S.M.S.G. Jr. High Math, Vol, I, Charter on non-metric geometry (drawing)

INTEGERS

The number system is extended to include the integers (Whole numbers and their opposites). Understanding is expanded as the application of the basic properties is emphsized. We do not recommend a formal development at this time nor do we expect complete mastery and rapid computation.

Topics Which May Be Studied

Whole numbers and their opposites.
Properties of the Integers.
Problems involving Integers.

Concepts

Integers are the natural numbers and their opposites and zero.
Properties for addition and multiplication extended to the integers.
There is closure under subtraction.
Additive inverse property: For each integer a, there is an integer b and such that $a + b = 0$

Skills

Locating and labeling points on a number line.
Determining the greater or lesser of two given integers.
Modest facility in performing operations: - Subtraction and division should be approached as inverse operations.
Informal and intuitive solotion of equations.
Informal and intuitive solution of eninequalities..

Application

Many familiar ideas involve measurement in "opposite directions".. Ideas from real life might include:

Steps forward and backward.
Above and below zero.
A business man may have losses as well as gains.
Yards gained or lost in a football game.
Below and above sea level.

Suggestions for Implementation

Operations in the set on integers can be developed by inductive methods which make use of demonstrations and an appeal to the intuition. The number line approach may be used. Examining patterns in wholes as they extend to the integers will help students see that the operations seem to give reasonable results.

EXAMPLE:

8 - 8 = 0	4.5 = +20	4.-3 = -12
8 - 7 = +1	4.4 = +16	3.-3 = - 9
8 - 6 = +2	4.3 = +12	2.-3 = - 6
8 - 5 = +3	4.2 = +8	1.-3 = - 3
8 - 4 = +4	4.1 = +4	0.-3 = 0
8 - 3 = +5	4.0 = 0	-1.-3 = + 3
8 - 2 = +6	4.-1 = -4	-2.-3 = + 6
8 - 1 = +7	4.-2 = -8	-3.-3 = + 9
8 - 0 = +8	4.-3 = 12	
8 - 1 = +9		
8 - 2 = +10		

In subtraction heavy emphasis should be on the basic idea given by: a-b = c if and only is c+b = a.

Suggestions for Implementation (cont.)

For example:

12 - 7 = 5 because 5 + 7 = 12
8 + 3 =11 because 11 - 3 = 8

Emphasis should be placed on designating signed numbers as either negative or positive to eliminate confusion with operations.

Helpful Materials and References

Number lines.
Thermometer.
Today's Mathematics (S.R.A) 1964 pages 343 - 350
Handbook for Elementary Mathematics Workshop (State of Ill.)
Modern Math Through Discovery I, Silver Burdett p. 264 - 314
Modern General Math - Addison Wesley p. 342-352
Modern School Mathematics - Structure and Method 7 - Dolciana, p. 499 - 520

The study of number systems different from the set of whole numbers can be used to review properties of numbers and to further develop and extend understandings of the following properties:

1. Uniquenses
2. Closure
3. Commutativity
4. Associativity
5. Identity element
6. Distributivity
7. Additive and multiplicative inverse

It is not necessary to have a seperate unit on math systems. Concepts of math systems arise naturally in many contexts and whenever appropriate they should be discussed. Care should be taken to develop the concept and not just introduce vocabulary.

Topics Which May Be Studied

Clock arithmetic
Modular arithmetic
Other arithmetic systems
Systems where elements are motions

Concepts

Uniqueness
Closure
Commutativity

Concepts (con't.)

Associativity
Distributivity
Systems may have different combinations of these properties
Systems may be man made

Skills

Constructing and using operation tables.
Using the inverse operation approach to solve subtraction and division problems in other systems.
Proofs for properties.

Applications

To lead the student to make discoveries concerning other systems-other systems do not always behave like the systems to which students have been exposed.

The children's game of Simon Says can be made into a mathematical system. A simple one consisting of just 4 elements could be made of the commands: S.S. about face, S.S. right face, S.S. left face, clap hands.

Students may work out tables for the operations of addition and multiplication on the set of the even and odd numbers.

Example:

t	E	O
E	E	O
O	O	O

Applications (con't.)

Point out to the students that they are dealing here with the natural numbers --- zero is not included.
Similarly, tables may be worked out. for the operations of addition and multiplication in the integer set.

Example:

In comparing the two systems, include the observation that these two 1 systems don't look alike but they do act alike: they are isomorphic.
First ask them to think of a number which has a remainder of one when you divided the number by 3. Then ask them for a number which has a remainder of zero when it is divided by 3; then for a number which has a remainder of 2 when divided by 3. The next question is "Under our usual process of division, do we ever get remainder of 3 or 4 when we divide by 3?" They readily agree that if we did we had really not done our division "correctly". Then have them write on their paper a list of the first six whole numbers which had a remainder of zero when you divide them by 3. Most of them forget zero. They get 3,6,9,12,15, and 18, but they have to be reminded that when they divide zero by 3 the remainder was zero. Then ask them to make a list of the first six whole numbers that have a remainder of one when you divide by 3 and a list of the first six whole numbers that had a remainder of 2 when divided by 3.

We then set out to investigate the three sets:

(o, 3, 6, 9, 12, 15 ...)
(1, 4, 7, 10, 13, 19...)
(2, 5, 8, 11, 14, 17...)

In order to have convenient names for each of these sets we decided that since each element in the first set had remainder zero when divided by 3 and each member of the second set had a remainder of 1 when you divide by 3, and there was a remainder of 2 in the third case, the numerals 0, 1, and 2 ought to be part of the names that we assign to these three sets. To distinguish the name of the set from the name of one of the elements of the set, we used the bar notation. 2

0 = (0, 3, 6, 9, 12 . . .)
1 = (1, 4, 7, 10, 13. . .)
2 = (2, 5, 8, 11, 14. . .)

Thus, zero-bar is the set of all counting numbers which have remainder zero when divided by 3, etc.

Now suppose we try to set up a mathematical system involving these three elements. Most systems we have seen so far have had two operations - addition and multiplication. How should we add $\overline{1}$ and $\overline{2}$? We discovered that whenever we took one number from $\overline{1}$ and a number from $\overline{2}$ and added these we always got a number in 0:

For example:

$$4 + 5 = 9 \quad \text{and} \quad 7 + 8 = 15$$

The other combinations were analyzed in the same manner and the results summarized in a table.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

A multiplication table may be developed in a similiar manner.

X	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Now that both operations are defined we can check to see what properties they have. Since there are only three elements in the set every possible case can be checked.

You could ask more capable students to explain why this always happens. Such a justification would incorporate some of these ideas:

$4 = 3 - 1 + 1$ and $14 = 3 - 4 + 2$ so

$4 + 14 \; (3 - 1 + 1) + (3 - 4 + 2) = (3 - 1 + 3 - 4) + (1 + 2) =$

$3 \; (1 + 4) + 3 = 3 - 5 + 3 = 3 \;\; (5 + 1) = 3 - 6$

Verbally, a number in $\overline{1}$ is a multiple of 3 plus remainder 1, a number in $\overline{2}$ is a multiple of 3 plus remainder 2. Adding, we have a multiple of 3 plus a remainder 3 and hence a multiple of 3 with remainder zero.

To see about commutativity, associativity, identity elements, multiplication distributing over addition and vice versa, and additive and multiplicative inverse.

Rather than spend a long time with this system it is instructive to develop another system of remainders resulting from dividing by 5. When the tables for modulo 5 ("modulo" comes from "modulus" meaning "yardstick") have been constructed one can check to see that the operations have the same properties as the system modulo 3.

+	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{0}$
$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{0}$	$\overline{1}$
$\overline{3}$	$\overline{3}$	$\overline{4}$	$\overline{0}$	$\overline{1}$	$\overline{2}$
$\overline{4}$	$\overline{4}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$

X	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{2}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{1}$	$\overline{3}$
$\overline{3}$	$\overline{0}$	$\overline{3}$	$\overline{1}$	$\overline{4}$	$\overline{2}$
$\overline{4}$	$\overline{0}$	$\overline{4}$	$\overline{3}$	$\overline{2}$	$\overline{1}$

Some time should be spent discussing subtraction and division in this system. A most instructive approach is to emphasize that to find $4 + 2$ we have to find a number to make this sentence true:

$\overline{3}$ X___ = $\overline{4}$. In other words $\overline{4} + 3 = 3$ because $\underline{\overline{3}}$ x $\overline{3} = 4$.

Similarly $\overline{4} - \overline{1} = \underline{\overline{3}}$ because $\underline{\overline{3}} + \overline{1} = \overline{4}$. In fact, we know

$\overline{2} - \overline{3} = \underline{4}$ because $\overline{4} + \overline{3} = \overline{2}$

This indicates that

$(0, \overline{1}, \overline{2}, \overline{3}, \overline{4})$

is closed under all four operations.

One of the reasons for discussion division in the system modulo five is to prepare for discussing some interesting properties of multiplication and division in the system module 6. The multiplication table is constructed in the same way as before.

$\overline{0}$ = (0, 6, 12, 18 . . .) $\overline{3}$ = (3, 9, 15, 21 . . .)
$\overline{1}$ = (1, 7, 13, 19 . . .) $\overline{4}$ = (4, 10, 16, 22. . .)
$\overline{2}$ = (2, 8, 14, 20 . . .) $\overline{5}$ = (5, 11, 17, 123 . .)

Thus, the multiplication table will have these six elements in it, $\overline{0}$, $\overline{1}$, $\overline{2}$, $\overline{3}$, $\overline{4}$, $\overline{5}$.

Now, what happens when we multiply $\overline{2}$ by $\overline{2}$? We go to the set which is labeled $\overline{2}$ and select two elements: 2 is one, 8 is another--- multiply 2 by 8 - we get 16 - look to see what set 16 is in - it turns out to be in 4, So $\overline{2}$ x $\overline{2}$ = 4.

What is $\overline{2}$ multiplied by $\overline{3}$? We'll take $\overline{2}$ out of 2 and 3 out of $\overline{3}$ and multiply - we get 6. Where is 6? It is in the set that has no remainder when you divide by 6. So $\overline{2}$ x $\overline{3}$ = 0.

Since 2 x 4 = 8 and 8 is in $\overline{2}$ we say $\overline{2}$ x $\overline{4}$ = $\overline{2}$.
Since 7 x 3 = 21 and 21 is in $\overline{3}$ we say $\overline{1}$ x $\overline{3}$ = $\overline{3}$.
Since 2 x 3 = 10 and 10 is in 4 we say $\overline{2}$ x $\overline{5}$ = $\overline{4}$, etc.

We get

x	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$
$\overline{2}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{0}$	$\overline{2}$	$\overline{4}$
$\overline{3}$	$\overline{0}$	$\overline{3}$	$\overline{0}$	$\overline{3}$	$\overline{0}$	$\overline{3}$
$\overline{4}$	$\overline{0}$	$\overline{4}$	$\overline{2}$	$\overline{0}$	$\overline{4}$	$\overline{2}$
$\overline{5}$	$\overline{0}$	$\overline{5}$	$\overline{4}$	$\overline{3}$	$\overline{2}$	$\overline{1}$

Many division problems need to be given and checked by using this table but there are a few which are especially important.

$\bar{3} + \bar{2} = ?$ $\quad$ $5 + \bar{3} = ?$ $\quad$ $\bar{0} + \bar{3} = ?$

$\bar{3} + \bar{3} = ?$ $\quad$ $4 + \bar{3} = ?$ $\quad$ $\bar{0} + \bar{2} = ?$

These examples illustrate some difficulties with division in this system. Some divisions have no solutions; some have several possible answers. It is false that

For each number x $\bar{0}$, there exists a number y such that x y = $\bar{1}$
i.e., there is no x such that $\bar{2}$ - x = $\bar{1}$.

It is also false that

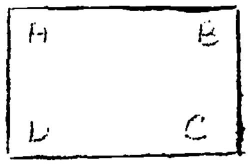
If ab = $\bar{0}$ then a = $\bar{0}$ or b = $\bar{0}$, since in particular $\bar{2} \times \bar{3} = \bar{0}$.

So here we have a system where some very familiar "facts" are not true; facts which are very important to ordinary arithmetic and algebra.

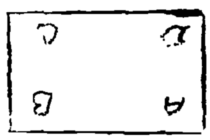
The SMSG text proceeds in a manner slightly different from what is outlined here. They have used physical models; an egg-timer clock or something of this sort that is familiar to the student to motivate the invention of these arithmetics. Still, the primary aim of the chapter is getting a fresh new context in which to discuss the commutative laws, the associative laws, the distributive law, closure, identic elements and so forth.

Another section of Chaper 12 concerns "Arithemetic without numbers".

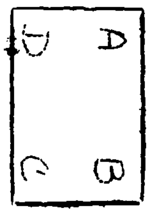
One such system can be obtained by considering a rectangle such as:



What are the motions which will put this rectangle in a position similar to the initial position without regard for position of letters? If it is turned 180° it will be in a position similar to the initial position:

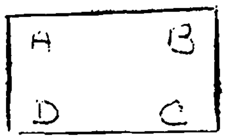


If it is turned only 90° , the rectangle is not oriented in the same fashion.

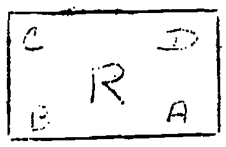


Let us identify the motions which will put this rectangle in a position similar to the one we started with. They are: a rotation of 180° - call this R: just leaving it alone - identity motion. I: a flip along the horizontal axis which we designate as a set of six motions I, R, X, V, and we can construct a table which shows us the f results of combining these various motions. What happens, for instance, when a flip on the horizontal axis is followed by a flip on the vertical axis? To ascertain the results of pairs of these motions it is helpful to record on the chalkboard the positions for each of these motions.

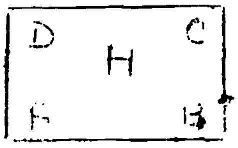
Starting with the card in this position



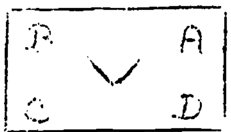
we will note the effect of motion R by writing



Motion H gives

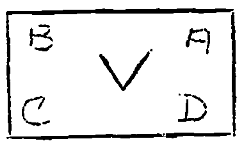


Motion V gives



2- Students will understand this motion more readily if each has a rectangular piece of cardboard about 4 inches by 6 inches at his desk.
See SMSG Commentary for lettering instructions.

In making an operation table we must agree to read it in a definite way; namely, a motion recorded in the left column followed by a motion listed in the top row. For example, a horizontal flip followed by a vertical flip results in a position which we could get by making the single motion R. Perform motion R and then perform motion H; the card is now in position.

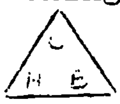


So R followed by H results in V. The completed table is below.

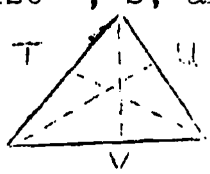
	I	R	H	V
I	I	R	H	V
R	R	I	V	H
H	H	V	I	R
V	V	H	R	I

With the table completed it is easy to check what properties the system has. It is commutative; associative; there is an identity element; each element is its own inverse.

You can do the same sort of thing with a Triangle such as this.



If you consider the set of motions consisting of: a rotation 60° clockwise, R; or a rotation 120°, S; and flipping it on the three altitudes as axes.



You get a system which is not commutative as the table below:

	I	R	S	T	U	V
I	I	R	S	T	U	V
R	R	S	I	U	V	T
S	S	I	R	V	T	U
T	T	V	U	I	S	R
U	U	T	V	R	I	S
V	V	U	T	S	R	I

U followed by R does not give the same result as R followed by U. However, the table does show that if only the motion I, R, and S are considered the result is a commutative system.

There is a lot of interesting mathematics which can be discussed in this chapter, but its main function should be to show that there are some systems that don't behave quite the way the systems of arithmetic and algebra do. It is significant to note that the University of Maryland Project, from whom SMSG borrowed this and many other chapters, includes this treatment of mathematical systems as the chapter following their discussion of properties of natural numbers. Thus, they felt it was a necessary part of the course and not just a cute but extraneous piece of mathematics.

MEASUREMENT

Measurement is one process that links the physical and social environment with mathematics. The concepts of measurement should be investigated at both the conscious and unconsious level in such simple situations as: "Will this fit?": "Have I time for this lesson?": "Is this board long enough?" To the technical level where measurement is regarded a process of estimating the magnitude of an unknown quantity with a fixed, relatively permanent standard, such as the yard or meter. Geometry is treated as a different topic within this Guide; therefore no mention is made of the geometric concepts associated with squared, circles, cubes, cones, etc.

Topics Which May Be Studied

- Capacity
- Time
- Money
- Length
- Area
- Volume
- Weight
- Comparison of English Units and Metric Units
- Temperature
- Using ruler to measure segments

Topics Which May Be Studied (con't.)

Precision and accuracy: interval of measure, precision, signific nt digits, round-off error, maximum possible round-off error, relative error, percent of error and accuracy.

Skills

Proficiency in using the tools of measurement, such as the ruler, stop-watch and scales..
Development of the reading of specific data and measurement content.
Estimating distances, areas and volumes.

Concepts

In the metric system of measurement the units used to measure a given property (length, capacity, weight) are related by some power of 10.

The english system of measures has no consistent pattern relating the units of measure of given property to one another. For example: 1 mile is 1760 yards or 3 x 1760 (5280) feet or 12 x 5280 (63,360) inches.

The smaller the unit of measurement the more precise the measurement.

The relative size of the various measuringunits such as square inch, square foot, square yard, cubic inch, cubic foot and cubic yard.

Applications

Indirect measurement, i.e., a measurement obtained from some other direct measurement. The height of a house is obtained indirectly from direct measurement of the height of the post and the length of two shadows. (See Trigonometry)

Computation with measurement numbers

Units of measure in ancient times

Applications (con't.)

Averages (See probability and statistics)

23 hour military time

Nautical Measures

How to read a micrometer

Other possible applications of measure: scale drawings, charts, graphs and tables.

Unit of measure and greatest possible error.

Pupils should compute actual mean (average) of the height and weight of their fellow classmates.

Implementation

Measure or weigh some object, first in the english system and then in the comparable metric system, i.e., quarts...liter.

Demonstrate the advantages of the metric system.

Pupil might gain a better notion of an acre if one could relate it to the size of the playground.

Nonstandard units, such as pace, surface covered by the hand, time taken to play phonograph record, a cupful of water would help develop concept of distance, area, time and capacity.

Discussion of "Where to Look" for various tables of measure would be of help in case pupil forgets.

One might introduce other special units of measurement and how they are used. Specifically recommended are: carot (wt. of precious stones), wire size or guage, nail size, horsepower, volt, watt and ampere.

Oral discussion as to appropriate unit of measurement for weight of coal, dimensions of room, distance between cities, distance of stars in light years, etc.

Implementation (con't.)

Emphasize the comparison aspect of measurement by using balance scale.

Pupils could practice estimating height and weight of their classmates. This could be followed by the actual use of appropriate measuring devices.

Determination of age in years, months, days could be included in work with denominate numbers.

Time zones and local time could be included in a unit in social studies.

(J. Houston Banks in writing about the Principles of Measurement tells us that situations from the pupil's experiences which involve measurement or aspects of measurement provide some of the best motivation for the introduction of mathematical processes.) This would seem to provide direction for those who are at a loss on how to introduce a new topic in mathematics.

NUMBER THEORY

Studying number theory is to introduce the students to many interesting and useful properties of sets of whole numbers. It is not expected that students memorize rules but rather that they explore patterns and relationships in the set of whole numbers both for fun and for use.

Topics Which May Be Studied

Whole numbers
Even numbers
Odd numbers
Factors
Multiples
Prime numbers
Composite numbers
Prime factorization
Greatest common factor
Least common multiple
Divisibility rules
Number patterns
Triangular, square and cubic numbers

Skills

Finding pairs of factors for numbers
Finding multiples of numbers
Finding prime factors of numbers
Finding greatest common factors
Finding least common multiples
Using the divisibility test to see how numbers may be factored
Recognizing patterns

Concepts

The set of whole numbers is the union of the set of odd and the set of even numbers.

Even numbers

Odd numbers

Every whole number except one has at least two different factors.

Every whole number is a factor of zero, but zero is not a factor of any non-zero number.

The set of all factors of a number is composed of all its pairs of factors.

Prime numbers have only one and themselves as factors or have exactly two factors.

A composite number is the product of two factors each smaller than the number itself.

Greatest common factor

Least Common multiple

We can determine the GCF or LCM of two numbers by finding the prime factorization of each number.

Divisibility rules for two, three, four and five.
4/last 2 digits = 4/number.

A triangular number is the sum of the consecutive whole number beginning with one.

Each square number greater than one is the sum of triangular number.

The sum of the cubes of consecutive whole numbers is the square of a triangular number.

Application

Students may construct factor trees when finding the prime factors of a number.

Students should use what they learn about number theory in working with rational numbers.

Application (con't.)

Use Goldbach's conjecture that every even number greater than two is the sum of two prime number.

Suggestions for Implementation

Use the Euclidean Algorithm as enrichment material for finding the greatest common factor.

Use the Sieve of Eratosthenes while studying prime numbers.

Number patterns are intended to stimulate interest in working with numbers.

Studying number theory is a good place for more practice in computation.

The study of many topics in number theory provides extra opportunity for practice. For example, the selecting for a Diophantiac equation such as $3m + 5n = 48$. (where m & n are whole numbers) can be found at this level by trial and error. This is both intriguing to the student and gives consideral computation practice.

Helpful Material and References

Introduction to Mathematics, Addison Wesley, pp. 92-100

New Goals in Mathematics, Van Nostrand, pp. 81-112.

School Mathematics I, Addison Wesley, pp. 137-147.

School Mathematics II, Addison Wesley, pp. 97-126.

NUMBERATION SYSTEMS

Numeration systems are studied to help develope a greater appreciation and understanding of the efficiency of a place value system, to clarify the decimal system by looking at similar and contrasting systems and to have fun. Numeration systems should be studied only to this extent. Mastery in computation is not desireable.

Topics Which May Be Studied

Ancient Numeration systems

Decimal system

Other-base systems

Expanded notation

Exponents

Skills

Limited skill in reading and writing of numerals using ancient system.

Using compact and expanded numerals of a base system.

Using exponents.

Concepts

Ancient numeration systems were mainly additive.

Base systems have place value.

Regrouping in computation.

Different ways of writing numerals.

Face-value, place-value, and total-value.

Expotential Notation.

Applications

Use of base systems in the 4 operations

Computers and base 2

Scientific notation, place-value, prime factors, and exponents.

Suggestions for Implementation

Comparison of decimal system with other base systems.

Development of addition, subtraction, multiplication and division tables in bases other than 10.

Using exponents in expressing the place-value of digits in a numeral.

Helpful Materials and References

Abacus

Place value charts

Charts of ancient systems

Other base charts

Number line

Napier's Rods

RATIO, PROPORTION & PERCENT

Instruction in this area should make clear that these ideas play an active role in daily adult, as well as, teen-age life. The logical progression from ratio to proportion to percent is strongly urged.

Topics Which May Be Studied

- Ratio
- Different names & expressions for ratios
- Proportions
- Proportions in solving problems
- Percent
- Percents greater than 100% & less than 1 percent
- Math statements using percent
- Use of proportion to solve percent problems
- % of increase
- % of decrease
- Discount
- Commission
- Interest - Simple
- Compound interest
- Interest formula

Skills

- Solving proportions
- Changing; to decimal form
- Writing proportions to solve proplems

Concepts

These 3 will be written out in sentence form to comply with format and meaning of ratio, proportion and percent.

The percent of increase vs. decrease

% is a special case in proportion

A ratio can be expressed in more than one manner

Applications

These concepts may be illustrated in the following areas through information and problem solving:

chemical solutions

recipes in cooking

banks and stores, stocks

graphs, sports statistics

geographical distances & facts - social population facts

distances in space, etc.

Implementation

To insure the understanding, use, physical demonstration where possible. The use of graphs to illustrate sizes and areas and also the use of how varying percents of a coloring agent effect the color of a whole solution is also possible.

General proportions such as $4/5 = 24/x$, $x/5 = 24/30$, etc. should be taught and used in problems before percent problems are encountered. When this is done there is little to do with percent and the three cases of percent or terms such as base rate and percentage need not and should not be mentioned. All percent problems can be solved using proportions such as $4/5 = x/100$, $4/x = 80/100$, and $x/.5 = 80/100$.

Helpful References and Materials

District resource center film catalogue
"Teaching Mathematics We Need" - Ginn Mod. Math. Series

RATIONAL NUMBERS

To establish a firm understanding of the positive rational numbers and the ability to use these. From this can be developed an understanding of the negative rational numbers and some ability to use them.

Topics Which May Be Studied

Define Set of Rational Numbers

Fractions

Properties of the Rationals

Equivalent fractions

Ordering of fractions

Four operations with fractions

Changing from fraction to decimal and vice versa

Decimals

Four operations with decimals

Repeating and terminating decimals

Extension of above to <u>all</u> of the rational numbers

Concepts

Order & Position of Rationals on number line

Fraction and decimal

Rationals may be represented as repeating and terminating decimals

Reciprocals

The extension of whole number properties to raise equivalency

Density

Skills

Graphing on number line
Reducing fractions
Performing the operations
Changing from fractions to decimals and vice versa
Solving equalities and inequalities
Computation with fractions and decimals

Applications

Ratio
Proportion
Remainder in division
Sets
Cooking
Measurement
Science
Social Studies
Percent

Implementation

Approaching this subject area as "rational numbers" places a responsibility to deal with negative numbers, etc. However, the traditional approach to this area as "fractions" relieves this responsibility and provides a much different viewpoint. It is suggested that the first approach to the subject matter allows the student to move in clear directions towards algebra, measurement, etc. in a real world sense. Hence, the implementation should be designed to provide the groundwork for this the movement into algebra by including the suggesting these new areas.

Helpful Materials and References

Rulers
Number lines
Fractional kits
Resource Center
"Decimals are easy" (film) Learning Resource Center
Fraction Series Filmstrips SVE

Meaning of Fractions

Changing the Terms

Adding Like Fractions

Adding Unlike Fractions

Subtracting Like Fractions

Subtracting Unlike Fractions

Multiplying Fractions

Dividing Fractions

REAL NUMBER SYSTEM

This area provides the students with an introduction to the set of real numbers. We don't propose to exhibit a complete list of properties for the real numbers, but students should see that there are many non-rational numbers on a number line. Properties which held in the set of whole, integers, and rationals are to be extended to the set of reals and indifferences illustrated.

Topics Which May Be Studied

Description of the set of real numbers.

Properties of the real numbers.

Rational approximations for irrational numbers.

Graphing irrational numbers.

Property of completeness.

Squares and square roots.

Approximating square roots.

Scientific notation.

Concepts

Decimal approximations for irrational numbers are non-repeating infinite decimals.

Rational approximations for irrational numbers.

Every real number corresponds to a point on the number line and every point on a number line corresponds to a real number.

There is an infinite number of irrational numbers.

Skills

Recognizes the basic properties of the real numbers.

Recognizes the completeness property of real numbers.

Graphing the set.

Using a square root table.

Applications

The student would now realize that we can now assign a number to each point on the number line.

Implementation

The set of rational numbers in union with the set of irrationals forms the set of raal numbers. Since all rationals can be named by repeating infinite decimals, we can define the irrationals as non-repeating infinite decimals. A review of the rational system will be needed to establish the meaning.

Previously, students have found solutions to inequalities by first finding the solution of a corresponding equation, and then deciding whether numbers $>$ or $<$ satisfy the inequality. An introduction to solving inequalities by using properties may be given at this time. You might state each property in terms of "greater than" and have the students varify that it is true.

Example: If a,b and c are any real number and $a > b$ then $a + c > b + c$.

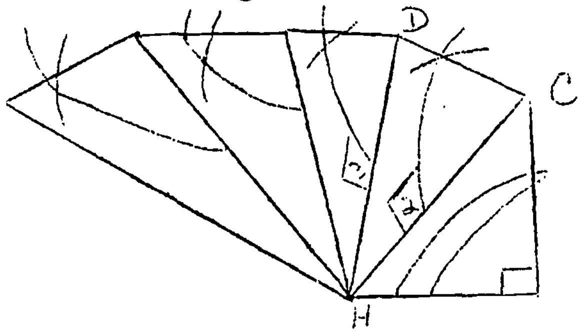
Many square roots are irrational numbers. When working with numbers like $\sqrt{3}$ and $\sqrt{11}$ we often use rational approximations for these numbers. Examine the real number line to see how the irrational numbers may be represented on it. Consider $\sqrt{2}$ which approximates 1.4142. It is between the rational numbers 1 and 2, and its limit can be approximated by a method such as the following:

$$1.4 < \sqrt{2} < 1.5$$
$$1.41 < \sqrt{2} < 1.42$$
$$1.414 < \sqrt{2} < 1.415$$

Students should realize the more precise the approximations the more precise we would be able to locate on the number line.

In working with the Pythagorean property, students can locate on the number line the points for irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ and so on.

The following "spiral" makes an interesting construction which is related to work with reals.



Construct a right triangle ABC with sides one inch long. The length of $\overline{AC}$ is $\sqrt{2}$. Now construct a similiar right triangle ACD. The length of $\overline{AD}$ is $\sqrt{3}$. As soon as the pattern is established have the students copy and extend the construction.

Helpful Materials & References

Extending Mathematics - 8 - Deans, Kane, McCeen, American Book Co., pp. 215-260

Modern School Mathematics - Structure and Method - Dolciani, Woeton, Berkenbach, Markert 8 - pp. 233-255

STATISTICS & PROBABILITY

A study of statistics should acquaint the student with those mathematical ideas and procedures used so often in material seen outside as well as inside the classroom; books, magazines and newspapers. It is well also to acquaint the student with those ideas used to classify data - often which concerns the students themselves. The work with probability should be fun for the student and its application should help them see that mathematics holds the key to some interesting problem solving.

Topics Which May Be Studied

- Collecting & listing data
- Statistical graphs
- Measures of central tendency
- Random sampling
- Combinations
- Meaning of Probability
- Independent events
- Exclusive events
- Lying with Statistics
- Histogram
- Normal curve

Skills

- Constructing graphs
- Organization of data
- Analyzing data
- Statistical inference

Concepts

The use and representation of data to aid a solution and clarify the normal curve.

Random sample or event.

The idea of outcomes as combinations.

Probability as a "likelihood".

The effect of independent & mutually exclusive events on the likelihood of success.

Applications

These two areas can be applied to any school area in which data can be recorded - shop, science, gym & school social studies. Graphical statistics occur in the texts and data is familiar. The application of the normal curve to science and nature is particularly interesting.

Implementation

The illustration of statistics is greatly aided by graphs & best shown to a class with prepared visuals-overhead projector etc. Differences & similarities can be pointed out by representing the same problem in different graphical methods. Emph enterpreting graphs, etc. de-emph making bar graphs, etc. should only assist in understanding. The dribbling of sand along an incline - illustrates the natural order of the normal curve.

Probability, on the other hand, is a bit more difficult but the use of dice, colored marbles is fun and instructive. The use of a deck of cards in this area should prove to be difficult as the numbers and combinations become difficult for some students.

Helpful References and Materials

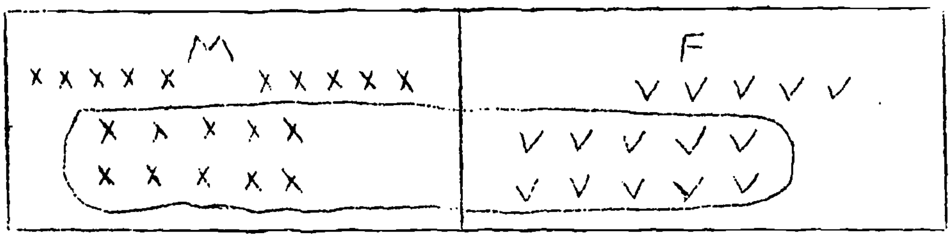
Jim, look at The Math Tchr in the last couple of yrs. for ideas

Also the N C T M yearbook on growth of Math Ideas.

Probability Machine, Addison Wesley, p. 349

School Mathematics II, Addison Wesley, p. 325-366.

Picture the following class in mathematics: 20 males and 15 females: 10 males wear glasses, and 10 females wear glasses.



Suppose we select an individual at random (with equal probability) from this class. Then we can find the probability of certain events (subsets). Some of these are given below.

Pr(M)	- 20/35	Pr (∧G)	- 10/35
Pr(F)	- 15/35	Pr(M ∨ F)	- 35/35
Pr(G)	- 20/35	Pr(M ∨ G)	- 30/35
Pr(M ∧ F)	-	Pr(F ∨ G)	- 25/35
Pr(M ∧ G)	- 10/35	Pr(G)	- 15/35
Pr(G / F)	- 10/15	Pr(G / M)	- 10/20
Pr(M / G)	- 10/20	Pr(F / G)	- 10/20

The problem above makes it clear that the language of sets is helpful in probability and statistics. Events are merely subsets of a universe. Many problems in life make use of the approach used here. We really have assumed a certain pattern in this study.

1. There is an operation of some kind - selecting an individual from a class.
2. There is a certain set of things that could happen--- in this case any person in class could be chosen.
3. There is a probability of each possible results--in our problem the chance of any person being selected is 1/35.
4. Each probability is equal or greater than zero and the sum of the probabilities is 1.

The probability of a certain event (subset) is always considered in relation to the entire event space. We ask the question "What could happen?" Consider throwing a die twice. The 36 things that could happen are pictured on the top of the next page of these notes.

B

H

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

The first digit is the number occuring on the first dice thrown, while the second digit is the number on the second dice. Finish the following questions about this even space.

QUESTIONS

1. The set in which the second element is number 1 is set__________.
2. The set in which the first number is 1 is set__________________.
3. The set in which both the numbers are the same is set__________.
(Use set notation of intersection, sum or complementation).
4. The set consisting of the element 1, 1 could be called in set language _________, _________, __________, or ___________.
5. The set consisting of the elements (1,1); (1,2); (1,3); (1,4); (1,5); (1,6); (2,2); (3,3); (4,4); (5,5); (6,6) is call in set language ________________________.
6. The probability of $A \wedge B$ = ________________________.
7. The probability of A = ____________________________.
8. The probability of B= ____________________________.
9. The probability of C = ____________________________.
10. The probability of $A \vee B$ = ________________________.
11. Can you think of some questions we could ask concerning the outcomes of this experiment of throwing two dice? For example, what is the probability of throwing a seven?

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PROBLEM

Let us consider a problem of winning on a lottery. To make it easier, we will say that on lottery I, there are five tickets to be sold and these tickets are numbered 1, 2, 3, 4, 5. A second lottery has four tickets to be sold and these are lettered a,b,c,d. We select a ticket from lottery I and then a tickey from lottery II. See if you can construct an event space of possible elements.

1. Note the possible results, things that could happen, that is, make an event space consisting of 20 different elements.

(1,a)	(1,b)	(1,c)	(1,d)
(2,a)	(2,b)	(2,c)	(2,d)
(3,a)	(3,b)	(3,c)	(3,d)
(4,a)	(4,b)	(4,c)	(4,d)
(5,a)	(5,b)	(5,c)	(5,d)

2. What elements are in set A if it consists of number 3 from lottery I, and we don't care what letter from lottery II.

   A = (3,a), (3,b), (b,c) (3,d)

3. What elements are in set B if it consists of letter B from lottery II, and we don't care what number from lottery I.

   B = (1,b), (2,b), (3,b), (4,b), (5,b)

4. Suppose that we say number 3 wins lottery I and letter b wins lottery II.

   a. What elements in the event space win lottery I; find the probability. Pr(A) = 4/20

   b. What elements in the complete event space (Universe) win lottery I and lose lottery II. Give this event a name and find its probability. Pr(A B') = Pr (3,a), (3,c), (3,d) = 3/20

   c. What elements in the complete even space (Universe) win lottery II and lose lottery I. Give this even a name and find its probability. Pr(B A') = 4/20

   d. What elements in the complete even space (Universe) lose lottery I and also lose lottery II. Give this event a name and find its probability. Pr (A' B') = 12/20

EXTENSION TO OTHER AREAS

This procedure of listing the possible results of an experiment leads to a complete event space for practical problems in many areas. These areas include industry, medicine, biology, and physics. Try to write possible results in a few examples to illustrate the method or principle involved.

1. A coin is tossed twice. What could happen?

   H,H   H,T

   T,H   T,T

2. A coin is tossed and a dice is thrown. What could happen?

   H,1   H,2   H,3   H,4   H,5   H,6

   T,1   T,2   T,3   T,4   T,5   T,6

3. A bag containing 3 white and 2 black balls is given. (a) One ball is drawn. (b) Two balls are drawn without replacement. Tell what could happen in both of these cases.

   W,W   W,B

   B,W   B,B

4. A class consists of 30 republicans and 20 democrats. Two of these students are selected at random, what could happen as far as choosing a democrat and a republican?

   R,R   R,D

   D,R   D,D

5. Two people go to work in the same building. They could enter any of four doors (N,E,S,W) with equal probability. What could happen?

   N,N   N,E   N,S   N,W

   E,N   E,E   E,S   E,W

   S,N   S,E   S,S   S,W

   W,N   W,E   W,S   W,W

The concepts, language, and symbols of set theory are not important in end of themselves at this level. The inclusion of these notions in the curriculum is solely for the clarification, simplification, and unification of other mathematical concepts. Understandings and skills should be developed with such utility constantly in mind.

Topics Which May Be Studied

Set recognition from description and roster

Element of a set

Set builder notation

Empty (null) set

Universal set

Disjoint sets

Subset

Equivalent sets

Equal sets

One-to-one correspondence

Union, intersection and complement of sets

Venn Diagrams

Cross (Cartesian) product

Properties of operations

Skills

Recognizing sets

Using set notations to designate sets

Constructing diagrams to illustrate union, intersection and disjoint sets

Concepts

Introducing concept of well defined set

Concept of symbol for subset and proper subset

Finding the number of subsets for any given set

Exploring concepts of finite and infinite sets

Applications

Using replacement sets and finding solution sets for equations and inequalities

Use of Venn diagrams for the union and intersection of sets in solution of verbal problems

Applying the concepts of union and intersection of sets to the solving of compound sentences with and or; applying the concept of set notation to the study of probability including sample, space, event, and simple events

Applying the concept of union to finding the probability of a compound event, mutually exculusive events

Applying the concept of intersection to independent events

Graphs of solution sets on inequalities in the universe of real numbers.

Equations and inequalities could be developed from story problems

Use of sentences involving "approximately equivalent to"

Graphs of function equations

Sets relating to Logic

Sets relating to LCM

Sets relating to GCF

Relations as subsets of cross products

Function as sets of ordered pairs

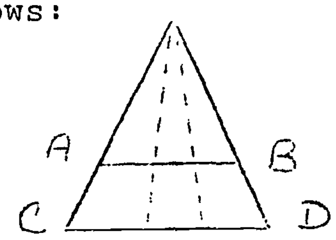
Suggestions for Implementation

Pupils should be constantly encouraged to express their mathematical ideas clearly, and in their own terms.

Stress continually that much of the language of mathematics is symbolic and that being able to use symbols and understanding their meaning is as fundamental to understanding mathematics as the ability to use and understand words.

For many pupils dealing with concrete objects will be easier to conceptualize than sets whose elements are abstract.

Continually review infinite and finite sets as some pupils have difficulty distinquishing between sets which are infinite and those which contain a very great number of elements. Illustrate that infinite sets are those that can be put into one-to-one correspondence with a proper subset of themselves by using: Whole numbers and even numbers (N 2N) and two segments A_______B and C________D as follows:



TRIGONOMETRY

Trigonometry is partly concerned with the measurement of triangles. At least this is our primary concern at the middle school level. Students should find the tangent, the sine, and the cosine of acute angles or right triangles by using the measures of the sides. The students should also learn to use a trigonometric table and solve problems involving indirect measurement.

Topics Which May Be Studied

Derivation of the word trigonometry.

Proportional relationships existing in similar right triangles.

Definitions of tangent, sine and cosine.

Trigonometric tables.

Indirect measurement.

Scale drawing.

Pyth, relation.

Concepts

Adjacent side and opposite side of acute angles.

Relation between tangents of acute angles of triangles.

Sine, cosine and tangent.

Skills

Writing and solving proportions.

Use of Formulas.

Using a trigonometric table.

Sketching triangles suggested by problems.

Making scale drawings.

Applications

The height of a flagpole, tree, and building may be found by using shadows.

The width of pond or river, navigation problems, airplane altitude may be investigated by using scale drawing or handmade surveying instruments and later perhaps by using professional equipment.

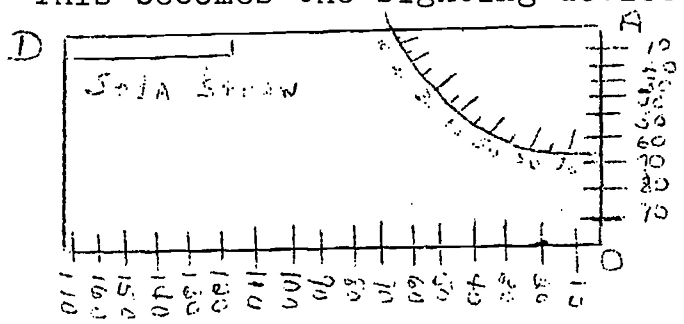
Suggestions for Implementation

These ideas might be developed as part of the geometry section on congruent triangles and similar triangles. Or, it might be part of an unit on measurement.
If it is possible to obtain surveying equipment you could have the students use indirect measurement to determine the height of such things as building or width of river. Less sophisticated equipment such as aldades and hypsometer planimeter can be made and reused. It must be made clear that development of the definitions of tangent, sine and cosine are necessary to the complete understanding of this subject area.

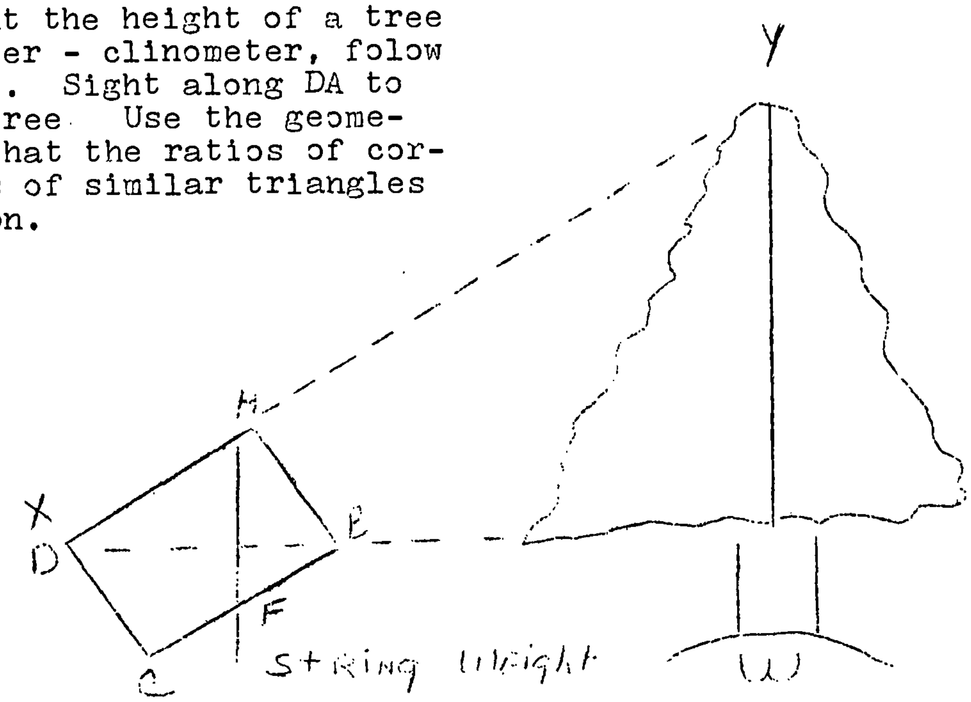
Tangents, sines and cosines can be expressed by fraction numbers as well as decimals.

Hypsemeter - Clinometer

Label the corners of a sheet of 10 squares to the inch-graph paper A,B,C and D in order. Mark 10,20,30,.....100 along the sides AB and BC as shown on the figure below. With A as a center, draw an arc of 90° with a radius of 5". Use a protractor and mark off the divisions along this arc 5°, 15°, ... 85°. Label the 10°, 20°, 30°,.... 80° divisions. Fasten the paper on the cardboard either using glue or masking tape. Using a needle insert a 12" string at A and fasten on the back side with masking tape or Scotch tape. On the front side, fasten a fishline sinker or some weight on the end of the string. Tape a soda straw along DA. This becomes the sighting device.



To figure out the height of a tree using a hypsometer - clinometer, folow these directions. Sight along DA to the top of the tree. Use the geometric principle that the ratios of corresponding sides of similar triangles are in proportion.



The angle of elevation is YXZ. Triangle ABF if similar to triangle YXZ. If AB = 100 and BF = 40 on the chart, and if the distance XZ is measured in feet by a tape, then ZY may be computed by using the proportion: AB, BF = XZ, XY

Add WZ to ZY to find the total height of the tree.

WHOLE NUMBERS

The study of whole numbers at this level is to provide the students with an understanding of the whole number system and their properties. Avoid excessive drill on computational skills since adequate practice can be provided while treating topics such as number theory and numberation systems. The role of word problems should be enlarged and emphasized as they provide the link between mathematics and the real world.

Topics Which May Be Studied

Natural numbers and zero

Infinite sets

Commutative property

Associative property

Distributive property

Additive identity element

Closure and uniqueness

Operations

Estimating products and quotients

Estimating solutions to problems

Number line graphing

Skills

Reasonable facility in performing the four basic processes.

Estimating sums, differences, products and quotients.

Graphing sets of whole numbers.

Concepts

Whole numbers consist of the set of natural numbers and zero.

The set of whole numbers is an infinite set.

The set of whole numbers is closed under addition and multiplication but not under subtraction and division.

Subtraction and division are neither associative nor commutative.

Uniqueness holds under all operations.

Division by zero is undefined.

Addition and subtraction are inverse operations and multiplication and division are inverse operations.

The set of whole numbers may be graphed on the number line.

Application

Solving real life story problems.

Graphing ordered pairs.

Graphing subsets for equations.

Suggestions for Implementation

Use set ideas and terminology when working with whole numbers.

Use various approaches to computational skills.

Emphasize mathematical structure.

Use games as drill.

Helpful Material and References

School Mathematics I, Addison Wesley, pp. 1-34.

School Mathematics II, Addison Wesley, pp. 35-70.

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